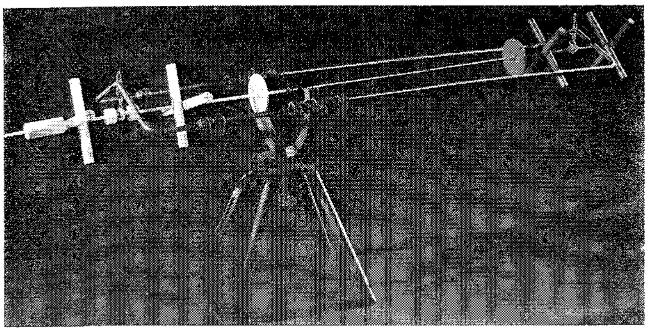

Fig. 5 Photograph of test assembly.

corresponding values for the combined system. Finally, certain pendulum modes of vibration of the model are possible since the short hanger/eyelet which suspends each mass from the frame permits small horizontal displacements. The fundamental frequencies due to these pendulum modes were detuned by adjusting the length of the hanger/eyelet assembly such that all pendulum frequencies were well above the natural frequencies under investigation.

Reference

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Deployment Dynamics of Rotating Cable-Connected Space Stations

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Nomenclature

$\mathbf{a}$ = acceleration of mass element
dm = mass of cable element
$d(t)$ = variable distance from center of mass to mass m_1
$\mathbf{F}_g$ = gravity gradient force
$\mathbf{F}_T$ = net tension force on cable element
G_e = gravitational constant
$\mathbf{i}$ = unit vector directed along rotating x axis
$\mathbf{j}$ = unit vector directed along rotating y axis
$\mathbf{k}$ = unit vector directed perpendicular to the xy plane
$l(t)$ = deployed length of the cable
L = total length of cable
m_1 = constant end mass which contains the undeployed cable
m_2 = constant end mass
$\mathbf{r}$ = position vector from Earth center to cable element
$\mathbf{R}_o$ = position vector from Earth center to system center of mass
t = time
T = cable tension
xyz = axis system rotating in orbital plane
x = coordinate directed along a line from m_1 to system mass center
y = coordinate directed perpendicular to x axis
ζ = distance from mass center to m_2, $(l - d)$
θ = angle from local horizontal to rotating x axis
$\dot{\theta}$ = angular velocity
$\ddot{\theta}$ = angular acceleration
$\boldsymbol{\varrho}$ = position vector from mass center to cable element
σ = dm/dx
Ω = orbital frequency

Received June 29, 1970; revision received August 7, 1970. This work was accomplished while the author was an ASEE-NASA Summer Faculty Fellow at Langley Research Center, Hampton, Va., in 1969. The author is indebted to A. P. Boresi, Professor of Theoretical and Applied Mechanics at the University of Illinois, for his helpful comments.

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Introduction

THE need to provide artificial gravity to crew members in a weightless environment is a vague physiological problem which has not yet been answered. There is some evidence that prolonged periods of weightlessness may cause irreparable damage to the human body and until it is shown that extended orbital stay times are harmless, no one is likely to insist that the artificial gravity capability be omitted. Accordingly, space stations of the future will provide an artificial gravity environment for the crew members. One means of providing artificial gravity is to rotate the space station while in orbit. The simplest design of such a system consists of a cable or a set of cables connecting two end masses, perhaps an Apollo spacecraft and experimental module with an S-IV rocket stage,[1] and may include a reel in, reel out capability. The basic feasibility of a rotating tethered system was investigated during the Gemini XI mission.[2]

Investigations of the deployment (reel in, reel out) of rotating cable-connected space stations[3] and the somewhat similar problem of the retrieval of a tethered inert mass[4] have been accomplished. These works, however, disregard the mass of the cable. Accordingly, no information is available concerning the waveform of the cable during deployment. Evidently, the inertia forces of the cable transmit a force input to each end mass, but the nature of the force is unknown until the waveform of the cable is known. As a result, the transverse motion of the cable during deployment could cause undesirable effects on the end masses. Moreover, knowing the waveform allows the tension to be calculated, and thus provides important cable design information. Accordingly, the deployment problem should be studied by considering the mass of the cable.

As a first step, this note derives the equations of motion for the cable during the deployment of a rotating cable-connected space station in Earth orbit.

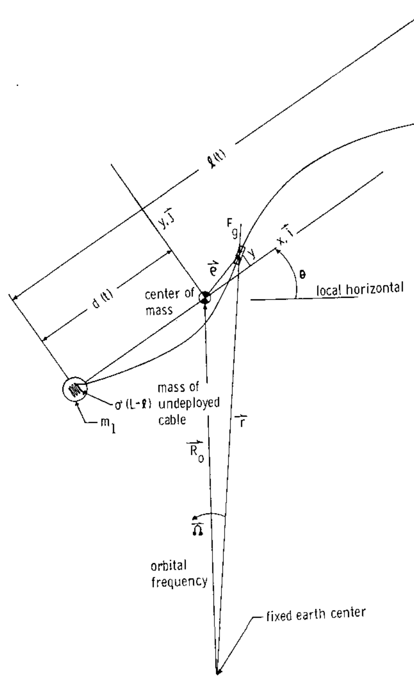

Fig. 1 Idealized model of space station system during deployment.

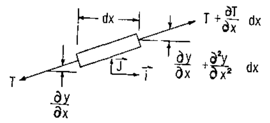

Fig. 2 Tension on differential cable element.

Development of the Equations of Motion

The model shown in Fig. 1 consists of two Earth-orbiting masses connected by a cable, rotating in the orbital plane about the system center of mass which moves in a circular orbit about the Earth. The cable is being deployed from one of the end masses by centrifugal force due to rotation and thus the separation distance between the end masses is increasing with time. Under these conditions, the assumption is made that the gravitation force $\mathbf{F}_g$ and the net tension force $\mathbf{F}_T$ are the only forces acting on a differential length of the cable. The gravity gradient force $\mathbf{F}_g$ is given by

$$\mathbf{F}_g = -G_e(\mathbf{r}/r^3)dm \tag{1}$$

where G_e is the gravitational constant, dm is the mass of the cable element, and $\mathbf{r}$ is the position vector from the center of the Earth to a cable element. The rotating coordinate system xyz with unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ has as its origin the center of mass of the system and rotates so that the unit vector $\mathbf{i}$ remains directed along the line from m_1 to the center of mass. The position vector $\mathbf{r}$ may be written $\mathbf{r} = \mathbf{R}_o + \boldsymbol{\rho}$, where $\mathbf{R}_o$ is the position vector from an inertial coordinate system considered fixed at the Earth's center and $\boldsymbol{\rho}$ is the position vector from the system mass center to a cable element. In terms of the rotating coordinate system, $\mathbf{r} = (R_o \sin\theta + x)\mathbf{i} + (R_o \cos\theta + y)\mathbf{j}$, where θ is the angle measured from the local horizontal to the x axis. Noting that $\rho \ll R_o$, $r^2 = R_o^2 + 2R_o\ (x \sin\theta + y \cos\theta)$, and from the binomial series, $(1/r^3) = (1/R_0^3)[1 - 3(x \sin\theta + y \cos\theta)/R_o]$. The gravitational constant can be obtained by equating the centripetal acceleration to the gravitational acceleration. Thus, if Ω is the orbital frequency, $G_e = R_o^3\Omega^2$. Using the aforementioned relations, and dropping higher-order terms, Eq. (1) may be written

$$\mathbf{F}_g = -[(R_o \sin\theta - 3x \sin^2\theta - 1.5y \sin 2\theta + x)\mathbf{i} - (R_o \cos\theta - 1.5x \sin 2\theta - 3y \cos^2\theta + y)\mathbf{j}]\Omega^2 dm \tag{2}$$

The net tension force $\mathbf{F}_T$ acting on the cable element can be obtained in the usual manner by referring to Fig. 2. Assuming small slopes and neglecting higher-order terms,

$$\mathbf{F}_T = (\partial T/\partial x)dx\mathbf{i} + [T(\partial^2 y/\partial x^2) + (\partial T/\partial x)(\partial y/\partial x)]dx\mathbf{j} \tag{3}$$

The acceleration of the cable element dm in terms of the rotating coordinate system is

$$\mathbf{a} = [-R_o\Omega^2 \sin\theta + \ddot{x} - \ddot{\theta}y - 2(\Omega + \dot{\theta})\dot{y} - (\Omega^2 + 2\Omega\dot{\theta} + \dot{\theta}^2)x]\mathbf{i} + [-R_o\Omega^2 \cos\theta + \ddot{y} + \ddot{\theta}x + 2(\Omega + \dot{\theta})\dot{x} - (\Omega^2 + 2\Omega\dot{\theta} + \dot{\theta}^2)y]\mathbf{j} \tag{4}$$

where $\dot{\theta}$ and $\ddot{\theta}$ are, respectively, the angular velocity and angular acceleration of the axis system with respect to the local horizontal and $\dot{\Omega}$ is taken to be zero.

The equations for the axial and transverse motion of the cable can be obtained from the vector equation

$$\mathbf{F}_g + \mathbf{F}_T = \mathbf{a}dm \tag{5}$$

Equating the coefficients of like unit vectors, two coupled nonlinear partial differential equations with variable coefficients are obtained.

$$(1/\sigma)\partial T/\partial x + (3\Omega^2 \sin^2\theta + 2\Omega\dot{\theta} + \dot{\theta}^2)x + (1.5\Omega^2 \sin 2\theta + \ddot{\theta})y + 2(\Omega + \dot{\theta})(\partial y/\partial t) = \ddot{x} \tag{6}$$

$$(1/\sigma)\partial/\partial x(T\partial y/\partial x) + (1.5\Omega^2 \sin 2\theta - \ddot{\theta})x + (3\Omega^2 \cos^2\theta + 2\Omega\dot{\theta} + \dot{\theta}^2)y - 2(\Omega + \dot{\theta})\dot{x} = \partial^2 y/\partial t^2 \tag{7}$$

In Eqs. (6) and (7), $\sigma = dm/dx$.

Expressions for $\dot{x}$ and $\ddot{x}$ can be obtained from Fig. 1 by noting that $\dot{x} = \dot{l} - \dot{d}$, where $\dot{l} = dl/dt$, etc. The deployment rate can be approximated by $\dot{l}$ and, since it can be controlled, is assumed to be a known function of time. From the definition of the mass center,

$$d = l(\sigma l/2 + m_2)/(m_1 + m_2 + \sigma L) \tag{8}$$

where d is the distance from the mass center to m_1, l is the deployed length of the cable, L is the total length of the cable, and m_1 and m_2 are the end masses. It follows that the expressions for $\dot{x}$ and $\ddot{x}$ are

$$\dot{x} = \dot{l}[m_1 + \sigma(L - l)]/(m_1 + m_2 + \sigma L) \tag{9}$$

$$\ddot{x} = [\ddot{l}(m_1 + \sigma L - \sigma l) - \sigma\dot{l}^2]/(m_1 + m_2 + \sigma L) \tag{10}$$

and are known functions of time. Equations (6, 7, 9, and 10) together with the relation between $\dot{\theta}$ and l provide the necessary equations to determine the waveform of the cable during deployment. It can be shown that the derived equations reduce to Targoff's[5] equations if $\dot{l}$ and $\ddot{l}$ are zero.

Discussion

The exact relation between $\dot{\theta}$ and l cannot be obtained without prior knowledge of the waveform $y(x,t)$ which in turn requires the relation between $\dot{\theta}$ and l. As a result, the exact determination of the waveform cannot be made. However, an approximate solution can be outlined. The simplest approach is to assume that orbital effects and the mass of the cable will have little effect on the relation between $\dot{\theta}$ and l. Then a direct application of the principle of conservation of angular momentum shows that $\dot{\theta}$ is independent of the end masses, and is given by

$$\dot{\theta} = \dot{\theta}_i l_i^2/l^2 \tag{11}$$

where $\dot{\theta}_i$ and l_i refer to the initial values of the relative rotation speed and the deployed length. An expression for the tension can be obtained from Eq. (6) by using the approach of Targoff[5] which neglects the effect of the vibration. Accordingly

$$T(x,t) \approx \sigma(3\Omega^2 \sin^2\theta + 2\Omega\dot{\theta} + \dot{\theta}^2) \times [(\zeta^2 - x^2)/2 + m_2\zeta/\sigma] - \sigma\ddot{x}(\zeta - x) - m_2\ddot{x} \tag{12}$$

where $\zeta = l - d$. Substitution of Eqs. (9–12) into Eq. (7) gives the basic differential equation for $y(x,t)$. Whereas Targoff[5] was able to separate variables and solve the resulting Legendre equation representing the spatial mode, and the Mathieu equation representing the temporal mode, separation of variables in the deployment problem is not possible and a numerical solution is required. The boundary condition is the basic differential equation evaluated at $x = l - d$ and $x = -d$, but with the term $\partial^2 y/\partial x^2$ omitted. The solution at any time must satisfy the center of mass definition

$$\sigma \int_{-d}^{l-d} y dx + m_2 y|_{l-d} = 0 \tag{13}$$

where $y|_{l-d}$ refers to y evaluated at $x = l - d$.

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